Statistical Interpolation of FET Data Base Measurements

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ABSTRACT

This work is the result of research into valid and compact statistical FET models. This paper presents a statistical interpolation technique which extends the Truth Model proposed by Purviance and Meehan in [6]. The Truth Model proposes to simply use samples from a FET measurement data base when performing statistical analysis and design of circuits.

The statistical interpolation technique presented here multiplies the number of points within a statistical data base by interpolating among the measurements in a statistically valid manner. It lends itself easily to software implementation, and gives results better than other simulation models now available.

We have developed and validated the statistical interpolation technique using 179 Galium Arsenide FETs supplied by TriQuint Semiconductor Inc. [5]. We show that the marginal statistics and the correlation matrix are preserved for the simulated samples.

1 Introduction

The past focus in statistical circuit design has been design for high parametric yield [1,2,7]. Yield is the fraction of circuits which meet specifications when the circuit parameters statistically vary around their nominal values. If good statistical models are used, software tools exist which can determine circuit designs for which parametric yield is in some sense optimized. To date, however, little work has been done to validate statistical component models, or the assumptions commonly made about the component statistics [6,5].

A FET is commonly characterized by a data base containing a number of measurements, n, of actual manufactured FETs. For instance, 179 FET measurements were used in this study. This data base then characterizes the statistical nature of the manufactured FET. The Truth Model [6] proposes to simply use the actual FET measurements when performing statistical analysis and design of circuits. This is practical and accurate when the number of measurements is "large".

$$
\begin{aligned}
S_i &= (\Re(S11(f_1)), \Im(S11(f_1)), \Re(S12(f_1)), \Im(S12(f_1)),\\
&\quad \Re(S21(f_1)), \Im(S21(f_1)), \Re(S22(f_1)), \Im(S22(f_1));\\
&\quad \vdots\\
&\quad \Re(S11(f_5)), \Im(S11(f_5)), \Re(S12(f_5)), \Im(S12(f_5)),\\
&\quad \Re(S21(f_5)), \Im(S21(f_5)), \Re(S22(f_5)), \Im(S22(f_5)))\\
S &= (S_1, S_2, \ldots, S_n)
\end{aligned}
\quad (1)
$$

Figure 1: Data format for the truth model

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In this paper, we develop a technique to increase the number of points in a FET data base in a statistically valid manner. This is accomplished by interpolating among the measured points. This "statistical interpolation" technique is developed in Section 4.

2 The Truth Model

The Truth model as applied to FET data [6] is essentially a means of using the actual measurements in simulating the FET performance statistics. During a Monte Carlo simulation of the circuit (for a good introduction to Monte Carlo simulation see [7]), a measured FET is chosen at random from the data base of the FET measurements, and the selected FET's S-parameters are used in a trial of the Monte Carlo Simulation.

The data for this study comes from Triquint Semiconductor Inc. and measures the 4 complex S-parameters of 179 GaAS FETs at 5 different frequencies. The measured data from each FET is stored as a vector with 40 components, i.e. 8 parameters $\times$ 5 frequencies = 40 total. The data format is illustrated in Figure 1, where S_i is a set of measured S-parameters for the i^{th} FET, and S is the entire set of measurements.

3 Statistical Interpolation

The object of statistical interpolation is to simulate, via the computer, sets of FET S-parameters which have the "same" statistics as the measured FET S-parameters which are contained in the FET's data base. An important point to address is what we mean by two S-parameter sets having the "same" statistics. To say two S-parameter data sets have the "same" statistics would mean comparing their 40 dimensional joint probability density function estimates. For this work, we will compare only the marginal densities and correlation matrices. This is consistent with the goals described in [5].

3.1 Statistical Interpolation Model

The technique we use for statistical interpolation is based on Kernel Density Estimation [3]. In Kernel Density Estimation, data samples are used as the basis for defining the shape of a probability density function (PDF) which is used to model the PDF of the process by which the original data was generated. Model parameters are chosen so that the PDF of the model "smooths" or interpolates the data, while simultaneously matching the statistics of the data PDF.

The model we used is based on the following equation:

$$\hat{S}_j = S_i + a\Delta S_j diag(K_i(k,h)) \tag{2}$$

Where:

$\hat{S}_j$	is the S-parameter vector generated from this model;
S_i	is a FET measurement vector chosen at random from the measured data;
a	is a constant model parameter;
ΔS_j	is a sample vector chosen at random from the Kernel PDF;
$K_i(k,h)$	is a scaling vector containing the distance from the chosen S_i to the kth nearest neighbor in each of the 40 dimensions [4].

The Kernel PDF is a 40-dimensional standard normal distribution with uncorrelated components and zero mean. The spread around each data point, "S_i", is determined by the model parameters "a" and "K_i". The sum of all the Kernel PDFs forms the PDF used to generate the simulated data.

The choice of parameters affects the smoothness of the simulated density function. For example, as seen in Figure 2 for a 1-dimensional model, too small of an "a" value will cause too much granularity in the simulated PDF whereas too large of an "a" value will cause the simulated PDF to be too smooth. The model parameters are chosen so that the correlation matrices, and the marginal densities of the measured and simulated match well. Analytical work shows that this model preserves the correlation matrix of the data only if the "a" variable is small. The validation in the next section demonstrates the accuracy of our model.

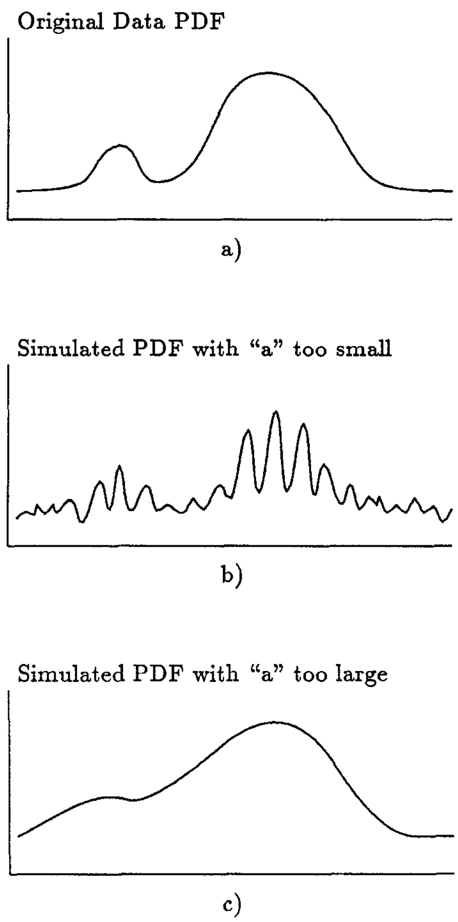

Figure 2: Density Functions for a 1-dimension example showing a) the original data PDF, b) the simulated PDF when the "a" constant is too small, and c) the simulated PDF when the "a" constant is too large.

4 Testing and Validation

The complete testing and validation of the model requires the verification that the probability density function of the simulated data points matches that of the measured data. We will show that the marginal densities and the correlation matrix of the simulated data match that of the measured data. The marginal densities are checked by the Kolmogorov-Smirnov test [6].

4.1 Marginal Densities

Figures 3 and 4 illustrate the model results with some example 2-dimensional scatter plots. These scatter plots were generated using the 179 Triquint FETs for the measured data, and 10,000 simulated FET measurements generated using our statistical interpolation model. The measured data is shown on the left and the simulated data is shown on the right.

Figure 5 shows the K-S probabilities for all the marginal densities for a typical run. The marginal densities for the simulated samples compare well to the marginal densities for the measured samples. All the K-S numbers are greater than 0.98 with most of them being greater than 0.999. This shows that there is a 98% – 99.9% probability that the marginal densities match.

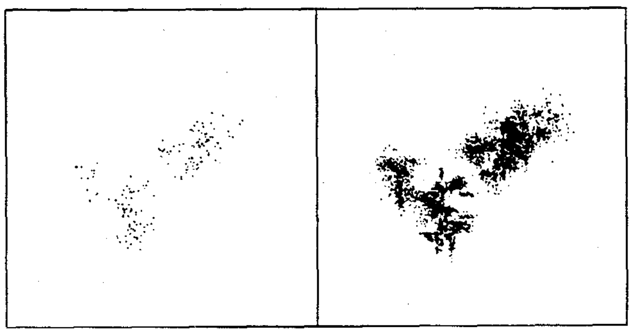

Figure 3: Real(S11) 6 GHz X-axis vs. Imag(S11) 6 GHz Y-axis

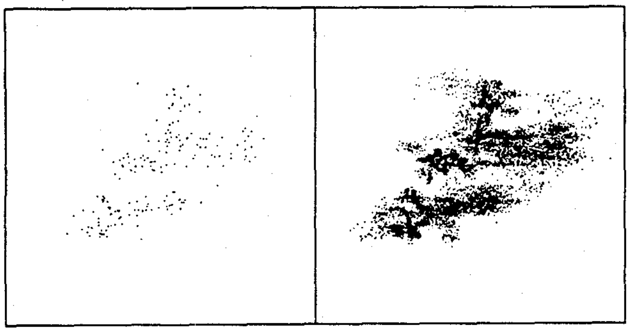

Figure 4: Imag(S12) 1 GHz X-axis vs. Real(S11) 26 GHz Y-axis

4.2 Correlation Matrices

The correlation matrices were compared using two methods: by computing the maximum difference between the elements of the matrices, and by the Euclidean norm of the difference matrix. The maximum difference was 0.06, and the Euclidean norm gave a value of 8.82×10^{-4}.

5 Conclusions

Statistical interpolation has been validated by comparing the marginal densities and the correlation matrix from the measured data set and a simulated data set. The comparison is excellent. The Kolmogorov-Smirnov test shows the marginal densities are statistically the same, and two-dimensional scatter plots further verify the model. The correlation matrices also compare well.

It should be noted, however, that while the Kolmogorov-Smirnov tests and the comparison of correlation matrices is the currently accepted method of comparing two multivariate densities, they are only a necessary but not sufficient test. More accurate tests are presented in [8].

The application of this model to statistical circuit design and analysis is presently under study. This model should also work well for FET model parameters. The authors are developing methods for choosing optimal model parameters for the statistical interpolation model.

Marginal	*KSnumber*
$\Re(S11(f_{1.0}))$	1.000000
$\Im(S11(f_{1.0}))$	1.000000
$\Re(S21(f_{1.0}))$	0.999998
$\Im(S21(f_{1.0}))$	1.000000
$\Re(S12(f_{1.0}))$	0.999959
$\Im(S12(f_{1.0}))$	0.992316
$\Re(S22(f_{1.0}))$	1.000000
$\Im(S22(f_{1.0}))$	0.999999
$\Re(S11(f_{6.0}))$	1.000000
$\Im(S11(f_{6.0}))$	0.999992
$\Re(S21(f_{6.0}))$	0.999902
$\Im(S21(f_{6.0}))$	0.999994
$\Re(S12(f_{6.0}))$	0.999825
$\Im(S12(f_{6.0}))$	0.999999
$\Re(S22(f_{6.0}))$	0.999813
$\Im(S22(f_{6.0}))$	0.991620
$\Re(S11(f_{13.5}))$	1.000000
$\Im(S11(f_{13.5}))$	0.999998
$\Re(S21(f_{13.5}))$	1.000000
$\Im(S21(f_{13.5}))$	1.000000
$\Re(S12(f_{13.5}))$	0.999998
$\Im(S12(f_{13.5}))$	0.999865
$\Re(S22(f_{13.5}))$	0.999998
$\Im(S22(f_{13.5}))$	0.999980
$\Re(S11(f_{21.0}))$	0.999998
$\Im(S11(f_{21.0}))$	0.999875
$\Re(S21(f_{21.0}))$	0.999989
$\Im(S21(f_{21.0}))$	0.999063
$\Re(S12(f_{21.0}))$	1.000000
$\Im(S12(f_{21.0}))$	0.999960
$\Re(S22(f_{21.0}))$	1.000000
$\Im(S22(f_{21.0}))$	0.999998
$\Re(S11(f_{26.0}))$	0.984350
$\Im(S11(f_{26.0}))$	0.999938
$\Re(S21(f_{26.0}))$	0.999999
$\Im(S21(f_{26.0}))$	0.999541
$\Re(S12(f_{26.0}))$	1.000000
$\Im(S12(f_{26.0}))$	0.999794
$\Re(S22(f_{26.0}))$	0.999979
$\Im(S22(f_{26.0}))$	0.999997

(3)

Figure 5: K-S numbers comparing measured and simulated marginal densities

Acknowledgements

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